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International Baccalaureate
Baccalauréat International
Bachillerato Internacional

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Wednesday 14 May 2014 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematical Studies SL*** *formula booklet* is required for this paper.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is *[90 marks]*.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. *[Maximum mark: 17]*

As part of his IB Biology field work, Barry was asked to measure the circumference of trees, in centimetres, that were growing at different distances, in metres, from a river bank. His results are summarized in the following table.

Distance, x (metres)	5	12	17	21	24	30	34	44	47
Circumference, y (centimetres)	82	76	70	68	67	60	62	50	50

(a) State whether *distance from the river bank* is a continuous **or** discrete variable. *[1]*

(b) **On graph paper**, draw a scatter diagram to show Barry's results. Use a scale of 1 cm to represent 5 m on the x-axis and 1 cm to represent 10 cm on the y-axis. *[4]*

(c) Write down

(i) the mean distance, $\bar{x}$, of the trees from the river bank;

(ii) the mean circumference, $\bar{y}$, of the trees. *[2]*

(d) Plot and label the point $\mathrm{M}(\bar{x}, \bar{y})$ on your graph. *[2]*

(e) Write down

(i) the Pearson's product–moment correlation coefficient, r, for Barry's results;

(ii) the equation of the regression line y on x, for Barry's results. *[4]*

(f) Draw the regression line y on x on your graph. *[2]*

(g) **Use the equation of the regression line** y on x to estimate the circumference of a tree that is 40 m from the river bank. *[2]*

2. *[Maximum mark: 14]*

A group of tourists went on safari to a game reserve. The game warden wanted to know how many of the tourists saw Leopard (L), Cheetah (C) or Rhino (R). The results are given as follows.

5 of the tourists saw all three
7 saw Leopard and Rhino
1 saw Cheetah and Leopard **but not** Rhino
4 saw Leopard **only**
3 saw Cheetah **only**
9 saw Rhino **only**

(a) Draw a Venn diagram to show this information. *[4]*

There were 25 tourists in the group and every tourist saw at least one of the three types of animal.

(b) Find the number of tourists that saw Cheetah and Rhino **but not** Leopard. *[2]*

(c) Calculate the probability that a tourist chosen at random from the group

(i) saw Leopard;

(ii) saw **only one** of the three types of animal;

(iii) saw **only** Leopard, given that he saw only one of the three types of animal. *[6]*

(d) If a tourist chosen at random from the group saw Leopard, find the probability that he also saw Cheetah. *[2]*

3. *[Maximum mark: 15]*

Consider the sequence $u_1, u_2, u_3, \ldots, u_n, \ldots$ where

$$u_1 = 600,\ u_2 = 617,\ u_3 = 634,\ u_4 = 651.$$

The sequence continues in the same manner.

(a) Find the value of u_{20}. *[3]*

(b) Find the sum of the first 10 terms of the sequence. *[3]*

Now consider the sequence $v_1, v_2, v_3, \ldots, v_n, \ldots$ where

$$v_1 = 3,\ v_2 = 6,\ v_3 = 12,\ v_4 = 24.$$

This sequence continues in the same manner.

(c) Find the exact value of v_{10}. *[3]*

(d) Find the sum of the first 8 terms of this sequence. *[3]*

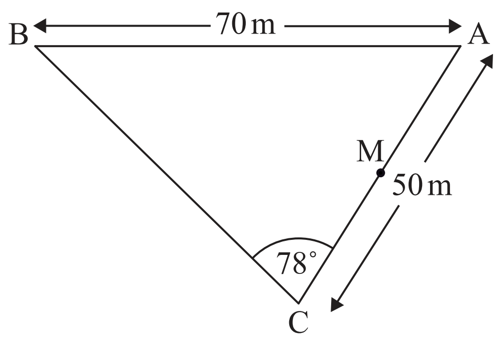
k is the smallest value of n for which v_n is greater than u_n.

(e) Calculate the value of k. *[3]*

4. *[Maximum mark: 15]*

ABC is a triangular field on horizontal ground. The lengths of AB and AC are 70 m and 50 m respectively. The size of angle BCA is 78°.

diagram not to scale



(a) Find the size of angle ABC. *[3]*

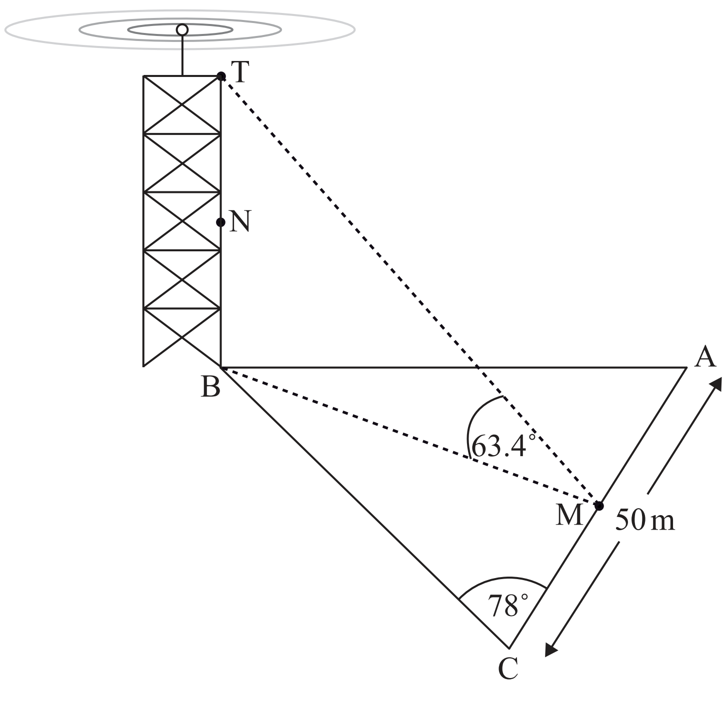
(b) Find the area of the triangular field. *[4]*

M is the midpoint of AC.

(c) Find the length of BM. *[3]*

A vertical mobile phone mast, TB, is built next to the field with its base at B. The angle of elevation of T from M is 63.4°. N is the midpoint of the mast.

diagram not to scale



(d) Calculate the angle of elevation of N from M. *[5]*

5. *[Maximum mark: 12]*

A group of candidates sat a Chemistry examination and a Physics examination. The candidates' marks in the Chemistry examination are normally distributed with a mean of 60 and a standard deviation of 12.

(a) Draw a diagram that shows this information. *[2]*

(b) Write down the probability that a randomly chosen candidate who sat the Chemistry examination scored at most 60 marks. *[1]*

Hee Jin scored 80 marks in the Chemistry examination.

(c) Find the probability that a randomly chosen candidate who sat the Chemistry examination scored **more** than Hee Jin. *[2]*

The candidates' marks in the Physics examination are normally distributed with a mean of 63 and a standard deviation of 10. Hee Jin also scored 80 marks in the Physics examination.

(d) Find the probability that a randomly chosen candidate who sat the Physics examination scored **less** than Hee Jin. *[2]*

(e) Determine whether Hee Jin's Physics mark, **compared to the other candidates**, is better than her mark in Chemistry. Give a reason for your answer. *[2]*

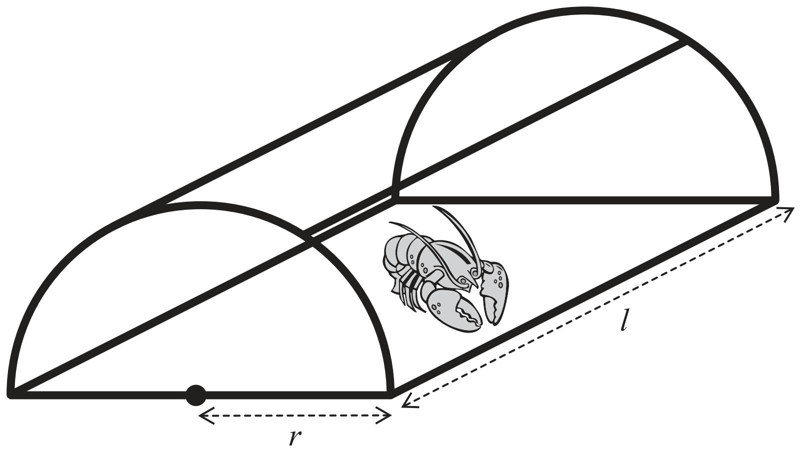
To obtain a "grade A" a candidate must be in the top 10% of the candidates who sat the Physics examination.

(f) Find the minimum possible mark to obtain a "grade A". Give your answer correct to the nearest integer. *[3]*

6. *[Maximum mark: 17]*

A lobster trap is made in the shape of half a cylinder. It is constructed from a steel frame with netting pulled tightly around it. The steel frame consists of a rectangular base, two semicircular ends and two further support rods, as shown in the following diagram.

diagram not to scale



The semicircular ends each have radius r and the support rods each have length l.
Let T be the total length of steel used in the frame of the lobster trap.

(a) Write down an expression for T in terms of r, l and π. *[3]*

The volume of the lobster trap is $0.75\,\text{m}^3$.

(b) Write down an equation for the volume of the lobster trap in terms of r, l and π. *[3]*

(c) Show that $T = (2\pi + 4)\,r + \dfrac{6}{\pi r^2}$. *[2]*

(d) Find $\dfrac{\mathrm{d}T}{\mathrm{d}r}$. *[3]*

The lobster trap is designed so that the length of steel used in its frame is a minimum.

(e) Show that the value of r for which T is a minimum is $0.719\,\text{m}$, correct to three significant figures. *[2]*

(f) Calculate the value of l for which T is a minimum. *[2]*

(g) Calculate the minimum value of T. *[2]*